Supporting Nested Resources in MrsP
Lemma 6 proof

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Lemma 6. The cost of each individual access (e') to a resource r^j is bounded by $e'^j = c^j + \sum_{r_k \in \mathbf{U}(r_j)} n_j^k e^k$.

Proof. As a consequence of Rule 1, there is at least one terminal resource r^t in the system not accessing any inner resource, i.e. $\mathbf{U}(r^t) = \emptyset$. For such a resource, its individual access cost is:

$$e'^t = c^t$$

From Lemma 5, if we simplify the queue of a resource as $q^j = |V(r^j)| + |map(G(r^j))|$ then the total access cost to e^t is:

$$e^t = q^t c^t \Rightarrow e^t = q^t e'^t$$

For the set of resources accessing the terminal resource, $\mathbf{V}(r^t)$, the individual access cost can be expressed as the execution time of the resource plus the access cost to its inner resource r^t as:

$$e'^{t+1} = c^{t+1} + n_{t+1}^t (q^t * c^t)$$

then substituting e^t:

$$e'^{t+1} = c^{t+1} + n_{t+1}^t e^t$$

And its total access cost can be again expressed as the queue for accessing the resource times the cost of accessing it.

$$e^{t+1} = q^{t+1}(c^{t+1} + n_{t+1}^t e^t)$$

For the 2nd iteration of t outer resources:

$$e'^{t+2} = c^{t+2} + n_{t+2}^{t+1}(q^{t+1}(c^{t+1} + n_{t+1}^t (q^t * c^t)))$$

$$e'^{t+2} = c^{t+2} + n_{t+2}^{t+1}(q^{t+1}(c^{t+1} + n_{t+1}^t e^t))$$

$$e'^{t+2} = c^{t+2} + n_{t+2}^{t+1}(e^{t+1})$$

Then, for the level k of nesting:

$$e'^{k} = c^{k} + n_{k}^{k-1}(q^{k-1}(c^{k-1} + n_{k-1}^{k-2}(...q^{t+1}(c^{t+1} + n_{t+1}^{t}(q^{t} * c^{t})))))$$

$$e'^{k} = c^{k} + n_{k}^{k-1}(q^{k-1}(c^{k-1} + n_{k-1}^{k-2}(...q^{t+1}(c^{t+1} + n_{t+1}^{t}(e^{t})))))$$

$$e'^{k} = c^{k} + n_{k}^{k-1}(q^{k-1}(c^{k-1} + n_{k-1}^{k-2}(...e^{t+1})))$$

$$...$$

$$e'^{k} = c^{k} + n_{k}^{k-1}(q^{k-1}(c^{k-1} + n_{k-1}^{k-2}e^{k+2}))$$

$$e'^{k} = c^{k} + n_{k}^{k-1}e^{k-1}$$

This can be directly applied to resources sequentially requiring more than one different independent inner resources:

$$e'^{k} = c^{k} + \sum_{r^{k-1} \in \mathbf{U}(r^{k})} n_{k}^{k-1}e^{k-1}$$