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Consolidation charts for non-linearly time-increasing loads

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Ground improvement techniques in the form of vertical drains combined with preloading are commonly employed to accelerate consolidation. In such situations, it is usually important to be able to consider the influence of the time needed for construction of the preloading on the time needed to achieve a certain degree of consolidation. Numerically computed charts for the design of vertical drains are presented in this paper, considering radial and vertical consolidation and non-linear schemes (i.e. parabolic, logarithmic) of the application of surcharge load increments against time. Examples are provided to illustrate how such charts can be employed to determine the required drain spacing in practical situations of preloading design. The influence of the type of preloading surcharge scheme is studied by means of a sensitivity analysis. Results indicate that the relative importance of radial consolidation with respect to vertical consolidation is the individual factor with the strongest influence on the results, and that drain spacing can be significantly increased when construction time is reduced. Results also indicate that the type of preloading scheme has a significant influence on the design of vertical drain spacing.

NOTATION

c_h	horizontal coefficient of consolidation
c_v	vertical coefficient of consolidation
E	total thickness of soft soil layer
H	maximum drainage distance in vertical direction
L	adimensional parameter indicating relative importance of radial to vertical consolidation
N	ratio r_e/r_d
r	distance in horizontal direction
r_d	radius of drain
r_e	maximum drainage distance in horizontal direction (i.e. equivalent drain spacing)
T	normalised time
T_c	normalised construction time
t	time
t_c	time for construction of surcharge
U	mean degree of consolidation
$\boldsymbol{u}$	excess pore water pressure
z	distance in vertical direction (depth)
$\boldsymbol{\sigma}$	total vertical pressure
$\boldsymbol{\sigma}_{\max}$	maximum vertical pressure imposed by surcharge

1. INTRODUCTION

The presence of soft, fine-grained soils with high compressibility is common in civil engineering projects, ranging from the foundations of buildings and embankments to the construction of fills in harbour areas. Ground improvement techniques in the form of preloading are commonly employed in such cases to anticipate and reduce settlements under future loads. In this context, the low permeability of soft soils often makes the time needed for dissipation of excess pore pressures unacceptable, and vertical drains combined with preloading are often required to ease radial drainage and accelerate consolidation. The time for construction of the preloading can represent a significant amount of the total construction time; however, the time for construction of the preloading is usually not considered in current practice, or it is considered only by means of approximate solutions. Therefore it is important to be able to consider the influence of the time for construction of the preloading on the time needed to achieve a target degree of consolidation so that, for instance, a structure can be founded on the soft soil producing an allowable settlement.

This papers builds on previous research (see References 1, 2), and it presents numerically computed charts for the design of vertical drains, considering radial and vertical consolidation and common non-linear schemes (i.e. parabolic, logarithmic) of the application of surcharge load against time increments. Such design charts can be employed to decide the 'optimal' spacing between vertical drains in practical applications. Finally, sensitivity analyses are also presented to study the influence of the type of surcharge on the time (or drain spacing) needed to achieve a specific degree of consolidation.

2. CONSOLIDATION WITH RADIAL DRAINS AND TIME-VARYING LOADS

Under the usual assumptions (see for example References 1, 3, 4), the dissipation of excess pore pressures within a layer of soft soil with vertical drains (considering joint radial and vertical consolidation as well as time-varying surcharge loads) is given by[1]

$$c_v \frac{\partial^2 \boldsymbol{u}}{\partial z^2} + c_h \left(\frac{\partial^2 \boldsymbol{u}}{\partial r^2} + \frac{1}{r} \frac{\partial \boldsymbol{u}}{\partial r} \right) + \frac{\partial \boldsymbol{\sigma}}{\partial t} = \frac{\partial \boldsymbol{u}}{\partial t} \tag{1}$$

where t indicates time, z represents depth, $\boldsymbol{\sigma}$ is the total vertical pressure due to the preloading surcharge, c_v and c_h are the vertical and horizontal coefficients of consolidation, and $\boldsymbol{u}$ is the excess water pressure with respect to hydrostatic conditions.

There is a wide variety of methods available for resolution of Equation 1 (for a review, see Reference 5). For instance, analytical solutions for the case of surcharge (ramp) loads that increase linearly during the time for construction of the preloading, t_c, have been derived considering common boundary conditions.[5] Such analytical solutions were then employed to develop charts for the design of vertical drains that can be used in cases of ramp loading conditions.[1]

This paper extends previous research to the case of surcharge loads that increase non-linearly with time. Specifically, it presents design charts for parabolic load increments of type $y = x^2$ (i.e. increasing from an initially zero rate to higher rates) and for logarithmic surcharge load increments of type $y = \ln(x+1)/\ln(2)$ (i.e. increasing from an initially infinite rate to lower rates); where $y = \boldsymbol{\sigma}/\boldsymbol{\sigma}_{\max}$ and $x = T/T_c$ (see Figure 1). Here $\boldsymbol{\sigma}_{\max}$ is the maximum vertical stress imposed by the surcharge, T is the (normalised) time, and T_c is the (normalised) construction time. Normalised times are computed as[1]

$$T = \left(\mu_1^2 + \frac{\pi^2}{4}L\right)\frac{c_h t}{r_d^2} \qquad (2)$$

$$T_c = \left(\mu_1^2 + \frac{\pi^2}{4}L\right)\frac{c_h t_c}{r_d^2} \qquad (3)$$

where t is the time, t_c is the construction time, and μ_1 is the first positive root of

$$Y_1(N\mu_1)J_0(\mu_1) - J_1(N\mu_1)Y_0(\mu_1) = 0 \qquad (4)$$

with $N = r_e/r_d$ being the ratio between the equivalent drain spacing and the actual radius of the drains. Tabulated solutions of Equation 4 are available.[5] Similarly, L is an adimensional parameter that quantifies the relative importance of radial consolidation compared with vertical consolidation (the lower the value of L, the more significant radial consolidation is). L is computed as

$$L = \frac{c_v r_d^2}{c_h H^2} \qquad (5)$$

where r_d is the drain radius and H is the vertical drainage distance.

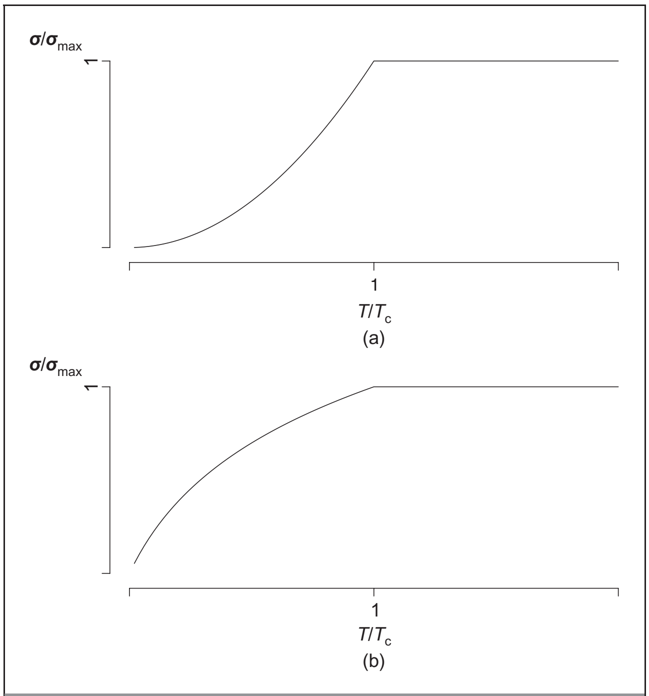

Figure 1. Non-linear evolution of surcharge load applied as a function of normalised time: (a) parabolic case; (b) logarithmic case

No analytical solutions are available for cases of non-linear load increments against time. Therefore superposition is used to compute excess pore pressures, as consolidation due to a surcharge increment is known to be independent of consolidation due to prior and posterior surcharge increments.[5,6] To that end, non-linear schemes of load increments against time are approximated as a series of linear (ramp) load increments, and excess pore pressures are computed as the sum of excess pore pressures corresponding to each ramp load increment.

3. DESIGN CHARTS

Design charts similar to those presented in Reference 1 have been developed for the case of a surcharge load that increases parabolically and logarithmically with (normalised) time. The influence of the ratio $N = r_e/r_d$ between the radius indicating the equivalent distance between drains and the radius of the drain itself, has been noted to be very small.[1] Therefore, for ease of comparison and in the interests of brevity, the results presented in this paper have been computed using the same values of N and $L = (c_v r_d^2)/(c_h H^2)$ as in Zhu and Yin's charts,[5] $N = 30$, and $L \in \{0{\cdot}5 \times 10^{-6}, 3 \times 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}\}$. Such design charts are shown in Figures 2 to 7 for the parabolic case and in Figures 8 to 13 for the logarithmic case. Figures 2 to 13 can be used for the design of vertical drain spacing using the methodology presented in Reference 1.

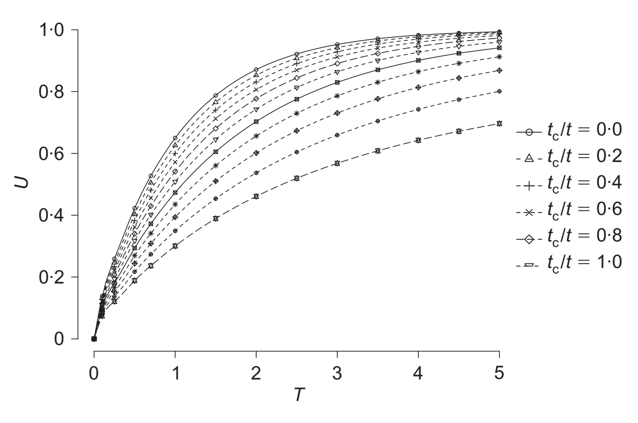

Figure 2. Mean degree of consolidation as a function of normalised time (parabolic surcharge; $L = 0$)

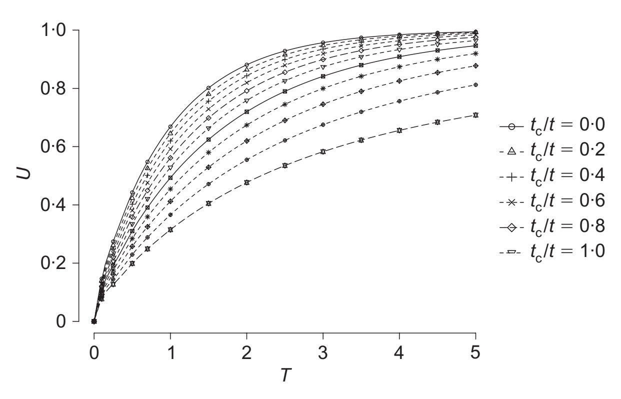

Figure 3. Mean degree of consolidation as a function of normalised time (parabolic surcharge; $L = 5 \times 10^{-6}$)

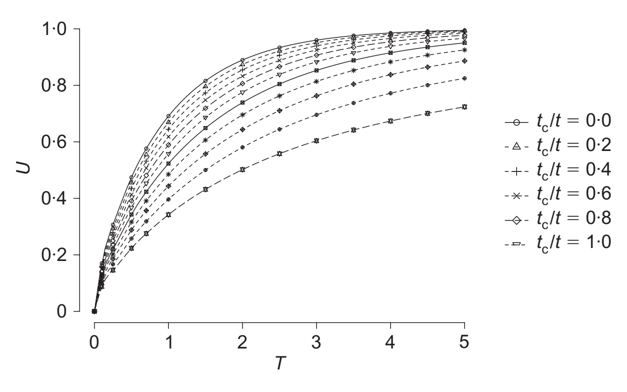

Figure 4. Mean degree of consolidation as a function of normalised time (parabolic surcharge; $L = 3 \times 10^{-5}$)

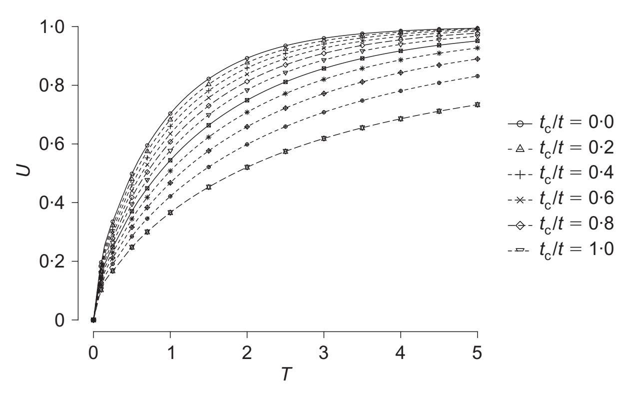

Figure 5. Mean degree of consolidation as a function of normalised time (parabolic surcharge; $L = 10^{-4}$)

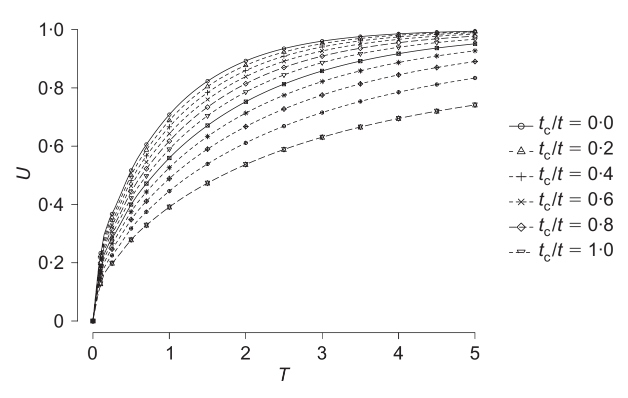

Figure 6. Mean degree of consolidation as a function of normalised time (parabolic surcharge; $L = 10^{-3}$

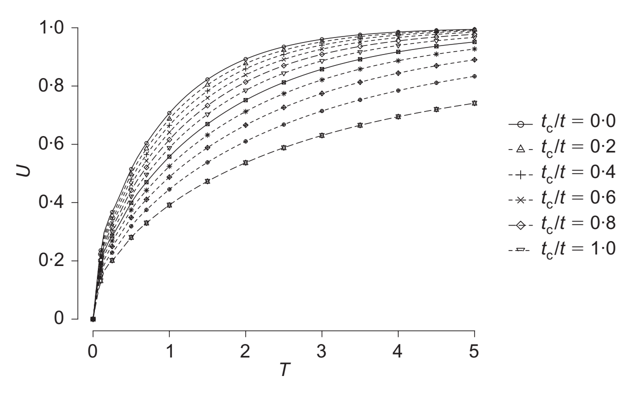

Figure 7. Mean degree of consolidation as a function of normalised time (parabolic surcharge; $L = 10^{-2}$)

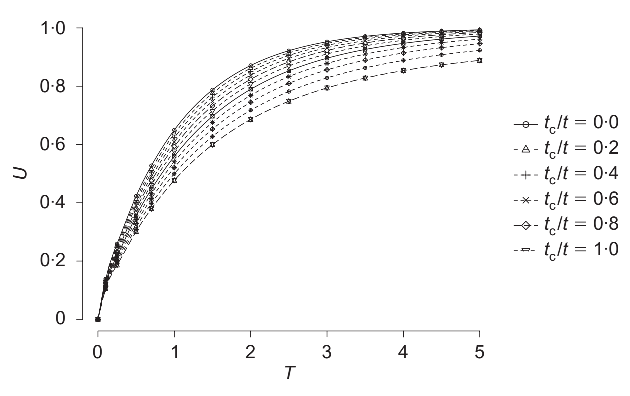

Figure 8. Mean degree of consolidation as a function of normalised time (logarithmic surcharge; $L = 0$)

4. SENSITIVITY ANALYSES

To study the influence of the type of surcharge load increment with time, the methodology discussed in Reference 1 is applied to a series of design cases. A clay layer with total thickness $E = 10{\cdot}0$ m and double-drainage conditions in the vertical direction are assumed. Therefore the maximum drainage distance in the vertical direction is $H = E/2 = 5{\cdot}0$ m. Drains available to be installed are assumed to have equivalent radius $r_d = 50$ mm, and the vertical coefficient of consolidation is assumed to be $c_v = 1{\cdot}5$ m^2/year in all cases. The value of the horizontal coefficient of consolidation c_h is modified to result in values of $L = 5 \times 10^{-6}$, $L = 10^{-4}$ and $L = 10^{-2}$, as summarised in Table 1.

For each type of surcharge increment with time and for each value of L considered in Table 1, four design examples have been solved, corresponding to two mean degrees of consolidation ($U = 0{\cdot}5$ and $U = 0{\cdot}8$), and to two values of the ratio between construction time for the surcharge and the time at which the target design value of U should be achieved ($t_c/t = 0{\cdot}4$ and $t_c/t = 0{\cdot}8$). In all cases it is assumed that the design

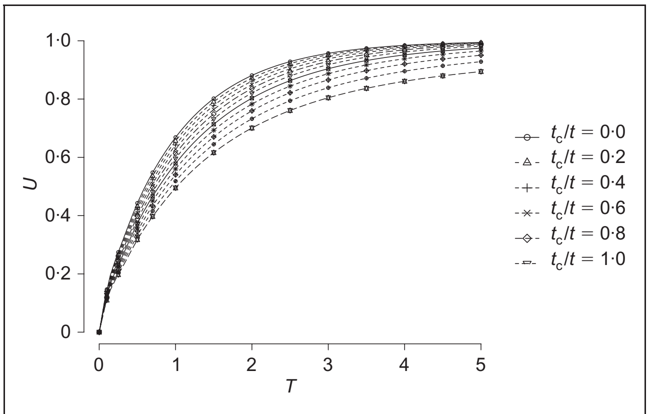

Figure 9. Mean degree of consolidation as a function of normalised time (logarithmic surcharge; $L = 5 \times 10^{-6}$)

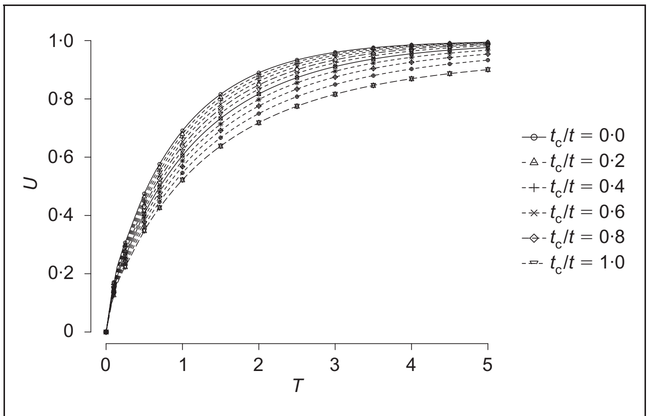

Figure 10. Mean degree of consolidation as a function of normalised time (logarithmic surcharge; $L = 3 \times 10^{-5}$)

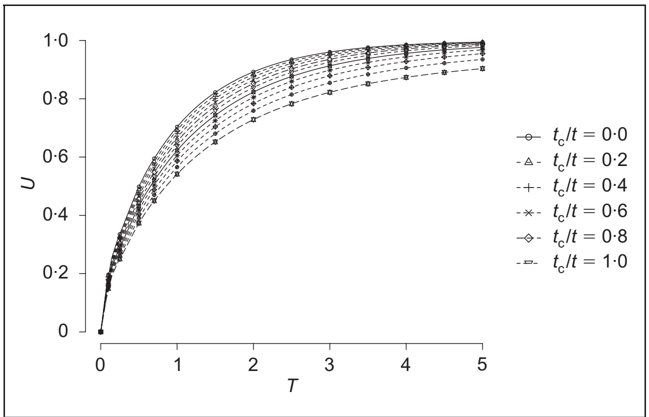

Figure 11. Mean degree of consolidation as a function of normalised time (logarithmic surcharge; $L = 10^{-4}$)

U values should be achieved after $t = 0{\cdot}3$ years. The values of T that correspond to the values of U considered for ramp, parabolic and logarithmic loading conditions have been listed in Table 2. Table 2 also shows the corresponding values of N, which are proportional to the spacing required between vertical drains to achieve the design target mean degree of consolidation.

Table 2 shows that the spacing with which vertical drains can be installed increases significantly as the value of L decreases.

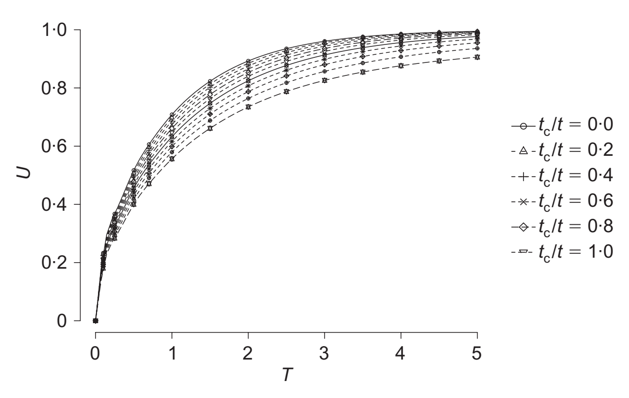

Figure 12. Mean degree of consolidation as a function of normalised time (logarithmic surcharge; $L = 10^{-3}$)

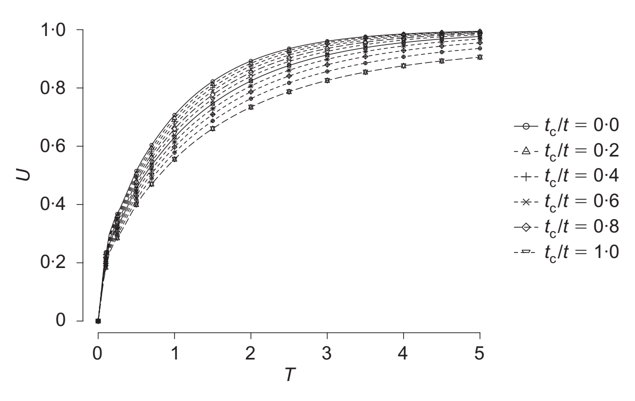

Figure 13. Mean degree of consolidation as a function of normalised time (logarithmic surcharge; $L = 10^{-2}$)

H: m	r_d: m	c_v: m^2/year	c_h: m^2/year	L
5·0	0·05	1·5	30·0	5×10^{-6}
5·0	0·05	1·5	1·5	10^{-4}
5·0	0·05	1·5	0·015	10^{-2}

Table 1. Design cases considered in the sensitivity analyses

This observation is expected, because the value of L is inversely proportional to c_h (remember that H, c_v and r_d are considered constant), and cases with wide variations of c_h values are considered, therefore introducing wide variations in the contribution of radial consolidation to the overall three-dimensional consolidation. In other words, L represents the relative importance of vertical consolidation with respect to radial consolidation: $L = 0$ implies horizontal flow and radial consolidation only, whereas $L \to \infty$ implies vertical consolidation only.[5]

Similarly, cases with equal values of L and equal load types have been considered to study the influence of the time of construction, t_c. As expected, results indicate that, everything else being equal, drain spacing can be increased as the speed of construction increases or, equivalently, the speed at which a certain mean degree of consolidation is achieved increases as t_c decreases. Specifically, for the example cases considered, the

Surcharge	L	U	t_c/t	T	N
Ramp	5×10^{-6}	0·6	0·4	1·040	48·07
Ramp	5×10^{-6}	0·6	0·8	1·490	41·16
Ramp	5×10^{-6}	0·8	0·4	1·900	37·10
Ramp	5×10^{-6}	0·8	0·8	2·830	31·33
Ramp	10^{-4}	0·6	0·4	0·881	14·87
Ramp	10^{-4}	0·6	0·8	1·260	12·73
Ramp	10^{-4}	0·8	0·4	1·755	11·04
Ramp	10^{-4}	0·8	0·8	2·627	9·30
Ramp	10^{-2}	0·6	0·4	0·881	2·19
Ramp	10^{-2}	0·6	0·8	1·235	1·89
Ramp	10^{-2}	0·8	0·4	1·740	1·63
Ramp	10^{-2}	0·8	0·8	2·599	1·38
Parabolic	5×10^{-6}	0·6	0·4	1·128	46·41
Parabolic	5×10^{-6}	0·6	0·8	1·887	37·21
Parabolic	5×10^{-6}	0·8	0·4	2·056	35·88
Parabolic	5×10^{-6}	0·8	0·8	3·624	28·23
Parabolic	10^{-4}	0·6	0·4	0·969	14·26
Parabolic	10^{-4}	0·6	0·8	1·624	11·41
Parabolic	10^{-4}	0·8	0·4	1·909	10·65
Parabolic	10^{-4}	0·8	0·8	3·326	8·42
Parabolic	10^{-2}	0·6	0·4	0·947	2·12
Parabolic	10^{-2}	0·6	0·8	1·570	1·71
Parabolic	10^{-2}	0·8	0·4	1·906	1·57
Parabolic	10^{-2}	0·8	0·8	3·313	1·24
Logarithmic	5×10^{-6}	0·6	0·4	0·965	49·66
Logarithmic	5×10^{-6}	0·6	0·8	1·201	45·16
Logarithmic	5×10^{-6}	0·8	0·4	1·755	38·38
Logarithmic	5×10^{-6}	0·8	0·8	2·313	34·13
Logarithmic	10^{-4}	0·6	0·4	0·849	15·11
Logarithmic	10^{-4}	0·6	0·8	1·050	13·77
Logarithmic	10^{-4}	0·8	0·4	1·658	11·31
Logarithmic	10^{-4}	0·8	0·8	2·132	10·16
Logarithmic	10^{-2}	0·6	0·4	0·818	2·26
Logarithmic	10^{-2}	0·6	0·8	0·978	2·09
Logarithmic	10^{-2}	0·8	0·4	1·658	1·67
Logarithmic	10^{-2}	0·8	0·8	2·113	1·50

Table 2. Values of T and N needed to achieve specified mean degrees of consolidation, U, for different surcharge load situations

drain spacing can be increased by up to 19% in the case of ramp loading, by up to 27% in the case of parabolic loading, and by up to 12% in the case of logarithmic loading, when the time of preloading construction is reduced to half (i.e. from $t_c/t = 0{\cdot}8$ to $t_c/t = 0{\cdot}4$).

It is also observed that, everything else being equal, surcharge load increments with a logarithmic shape allow larger vertical drain spacing (up to 11%) than surcharge load increments that are linear with time. Similarly, the scheme of linear surcharge load is observed to allow larger drain spacing (up to 10%) than the parabolic scheme. Separation of vertical drains can be up to 21% larger when the logarithmic preloading scheme is compared with the parabolic preloading scheme. Also, the difference between computed spacings for different preloading schemes is larger for high values of the mean degree of consolidation, U. This indicates that the differences between required vertical drain spacings could be increased if a higher value of U (say $U = 0{\cdot}95$) had been specified as design target.

Finally, in cases of excessive preloading speed there could be problems associated with instability of the soft foundation soil on which the surcharge fill is constructed. The possibility of occurrence of such instabilities, however, has not been considered in the results presented herein.

5. CONCLUSIONS

This work studies the problem of consolidation under surcharge loads that increase non-linearly with time. The influence of (normalised) time and (normalised) construction time on the evolution of consolidation is studied for schemes of surcharge increments in time of parabolic and logarithmic shape. Design charts that can be used for design of vertical drain spacing have also been developed, and sensitivity analyses to illustrate the influence of different design parameters (L, t_c and type of preloading) in practical cases of preloading design have been presented.

Based on the computed results, the following conclusions can be drawn.

(*a*) The spacing with which vertical drains can be installed increases significantly as the contribution of radial consolidation increases (i.e. as the value of L decreases). In fact, results indicate that L is the individual factor with the strongest influence on the computed vertical drain spacing.

(*b*) As expected, if aspects related to instability of the soft foundation soil are not considered, the earlier the preloading surcharge is applied, the larger the spacing with which vertical drains can be installed. In other words, consolidation is faster as more load is applied at early stages of the surcharge process. For instance, for the case of parabolic preloading, the spacing between vertical drains can be increased by up to 27% if the time for construction of the surcharge is reduced to half. (The reduction can be up to 19% for a ramp load and up to 12% for a logarithmic load.)

(*c*) Results also show that the type of preloading surcharge increments with time significantly affects the computed spacing between vertical drains. In that sense, the following can be observed, everything else being equal.
  - (i) The scheme of logarithmic preloading surcharge allows vertical drain spacings up to 11% larger than spacings computed considering the scheme of preloading surcharge that increases linearly with time.
  - (ii) The scheme of surcharge loads increasing linearly with time allows vertical drain spacings up to 10% larger than schemes of surcharge loads of parabolic type.
  - (iii) Separation of vertical drains can be up to 21% larger when the logarithmic preloading scheme is compared with the parabolic preloading scheme.
  - (iv) The difference between different preloading types (ramp, parabolic, logarithmic) is larger for higher values of the mean degree of consolidation than for lower values of the mean degree of consolidation (e.g. $U = 0{\cdot}8$ compared with $U = 0{\cdot}6$).

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